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ENERGY APPROACH TO THE SOLUTION OF THE NONLINEAR BOUNDARY-VALUE PROBLEMS OF THE NONCLASSICAL THEORY OF ANISOTROPIC STRATIFIED SHELLS WITH SLIT-CRACKS

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Unlike [1, 2], the energy approach proposed in [1] is used in the present paper to solve very nonlinear boundary-value problems of the nonclassical theory of anisotropic stratified shells containing slit-cracks. Three types of nonlinearity are considered: geometrical, physical, and structural. The third type of nonlinearity occurs due to a change in the theoretical scheme: when an external load acts on a shell with a slit-crack, contact may occur between the sides of the slit in the compression zone, and hence one must determine its effect on the stress-intensity factor. Stratified mildly sloping shells, weakened by one or two collinear rectilinear through-cracks are investigated. The main requirements imposed on the packet of layers are described in [1]. The behavior of such shells with slit-cracks is described by the Timoshenko-type theory taking into account the geometrical and physical nonlinearities. It is suggested that at all points of the body of the shell a process of active deformation occurs for simple loading with isotropic strengthening [3-5].

1. Formulation of the Problem. We will formulate the variational problem of the statics of the triply nonlinear nonclassical Timoshenko-type theory of an orthotropic stratified mildly sloping shell containing a slit-crack of length 2L situated along the X_1 axis. A Cartesian system of coordinates X_i ($i = 1, 2, 3$), the origin of which is situated at the center of the slit, relates to the coordinate surface of the shell. The axes of coordinates coincide with the axes of orthotropy of all the layers. An external load of strength q_n acts on the surface of the shell with the slit-crack.

We will formulate the nonlinear variational problem as follows. It is required to determine the steady value of the nonlinear functional V, which expresses the total or additional work of the shell with the slit-crack, starting from the condition that the first variation $\delta V = 0$ over the whole independent variable functional arguments, which satisfy the kinematic boundary conditions $u_{nj} = u_{nj}^0$, $\varphi_{nt} = \varphi_{nt}^0$ or the static boundary conditions $N_{njk} = N_{njk}^0$, $M_{njk} = M_{njk}^0$ ($j, k = 1, 2, 3$; $t = 2, 3$) or the mixed inhomogeneous boundary conditions on the contour of the shell for the whole packet [6], and also the boundary condition on the surface of the slit" on the part of the contact $|x_1| \le L$, $h_0 \le x_3 \le h/2$ for $x_2 = 0$, $N_{njk}^+ = N_{njk}^-$, $M^+_{njk} = M^-_{njk}$, $v^+ - v^- \ge c$, and on the surface of the slit, free from contact stresses, $N_{njk} = 0$, $M_{njk} = 0$ (h_0 is the distance from the coordinate surface to the origin of the contact part, h is the thickness of the shell, $N_{njk}^\pm$, $M_{njk}^\pm$ are the compressive, shear, intersecting, bending, and twisting contact forces and moments, $v^\pm$ are the displacements of points of the contour of the slit of the contact region, and c is the initial opening of the slit).

The local stress-strain state which occurs directly at the slit-crack is described by a nonlinear variational problem for an "infinite" stratified orthotropic shell containing this slit-crack, which is as follows: it is required to determine the steady state of the nonlinear functional V which expresses the total or additional energy of the shell with the slit-crack; the extremals must satisfy the boundary conditions on the surface of the crack, namely, on the part of the contact $|x_1| \le L$, $h_0 \le x_3 \le h/2$ for $x_2 = 0$ $N_{njk} = N_{njk}^1 + N_{njk}^+$, $M_{njk} = M_{njk}^1 + M_{njk}^+$ at the edges of the slit, free from contact stresses $N_{njk} = N_{njk}^1$, $M_{njk} = M_{njk}^1$ (N_{njk}^1 and M_{njk}^1 are the longitudinal, shear, and transverse forces, the bending and twisting moments for the whole packet, taken from the solution for a shell without a crack at the point where it is assumed the crack will occur), and also the conditions at "infinity": $u_{nj}|_{\rho\to\infty} \to 0$, $\varphi_{nt}|_{\rho\to\infty} \to 0$ (u_{nj} is the displacement of the points of the coordinate surface in the direction of the X_i axes, respectively, φ_{nt} are the angles of rotation of the normal to the median surface in the X_1X_3 and X_2X_3 planes, and ρ is the distance from the tip of the slit.

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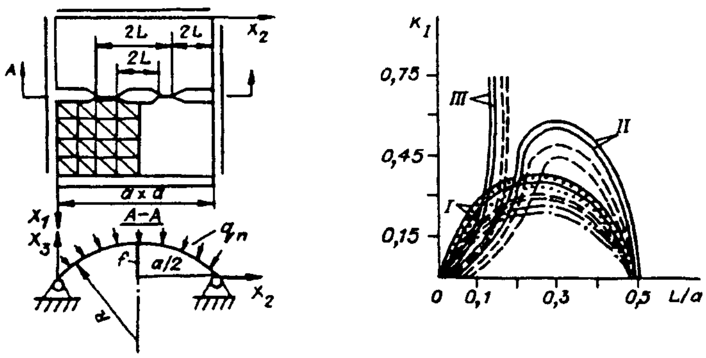

Fig. 1

An important factor in the highly nonlinear theory considered is the choice of the method of representing the functional relationship $\sigma_i = \varphi(e_i)$ (σ_i is the intensity of the stress and e_i is the intensity of the strain), which approximates the indicator curve of the material for simple stretching or shear and which satisfies the condition that it is independent of the form of the stress state. In this paper we will assume that the material of the shells behaves in the same way for stretching and compression, i.e., the function $\varphi(e_i)$ must be odd. The relationship used in the present paper is therefore a cubic spline-function [7-8].

The Mises plastic-state conditions [4] and Hill's energy theory of hardening [5] are used to solve problems of the theory of shells with slits. These enable one to carry out calculations on a material with isotropic nonlinear strengthening.

The solution of this nonlinear variational problem reduces to determining the value of the parameters which define the local fracture.

2. Method of Solution. In this paper we will consider the highly nonlinear problem with a symmetrical distribution of the stresses around the tip of the slit, characterizing the intensity of the energy G_I liberated and the stress intensity factor K_I for normal cleavage [1, 2, 9, 10].

The intensity G_I and the factor K_I were calculated using an energy approach, described by the following formulas:

$$V = \iint_S \int_0^r q(\mathbf{r})\, dr dS,$$

$$G_I = \partial V/\partial L, \quad K_I^2 = G_I \Big/ \left\{ \left(\frac{a_{11}a_{22}}{2}\right)^{1/2} \left[\left(\frac{a_{22}}{a_{11}}\right)^{1/2} + \frac{2a_{12} + a_{66}}{2a_{11}} \right]^{1/2} \right\}, \tag{2.1}$$

where V is the potential energy of nonlinear strain of the shell with a slit, which is the area bounded by the load-displacement diagram, and calculated using Simpson's formula, q(**r**) is a function which describes the load-displacement curve, **r** is the vector of the displacements of the point of the median surface of the shell with the slit, S is the surface of the shell with the slit, which is under load

$$a_{11} = \sum_{i=1}^{n} a_{11}^i, \; a_{11}^i = 1/E_{x_1}^i; \; a_{22} = \sum_{i=1}^{n} a_{22}^i, \; a_{22}^i = 1/E_{x_2}^i; \; a_{12} = \sum_{i=1}^{n} a_{12}^i,$$

$$a_{12}^i = -\nu_{x_1x_2}^i/E_{x_1}^i; \; a_{66} = \sum_{i=1}^{n} a_{66}^i, \; a_{66}^i = 1/\mu_{x_1x_2}^i; \; E_{x_1}^i, E_{x_2}^i$$

are Young's moduli of the layers, $\mu_{x_1x_2}{}^i$ is the shear modulus of the layers, and $\nu_{x_1x_2}{}^i$ is Poisson's ratio of the layers. Relations for the elastic constants can be written using the expressions given in [6]. Formulas (2.1) were derived for shells with slits having a thickness $h = 1$.

For physically nonlinear problems of the Timoshenko-type nonclassical theory of anisotropic stratified shells with slit-cracks the relations between the intensity G_I and the factor K_I, represented in (2.1), taking [9] into account, can be derived as follows: we use the hypothesis that the plastic deformations, which appear in the surface layer of the last layer of the shell in the region of the tip of the crack locally on the stretched side, relate to the case of small-scale plastic flow and are localized in a narrow strip along the line along which a crack of zero thickness develops and which takes into account the line of discontinuity of the elastic displacement; the layer of the shell with the crack considered behaves in the same way as a plate in a state of uniform tension. The value of the elastic displacements is then found by solving the linear problems of the theory of elasticity [1].

TABLE 1

$E^i_{x_1}/E^i_{x_2}$	K_I	$E^i_{x_1}/E^i_{x_2}$	K_I
2	0,407 0,428	20	0,399 0,418
5	0,403 0,426	40	0,399 0,426
10	0,404 0,428	60	0,408 0,420

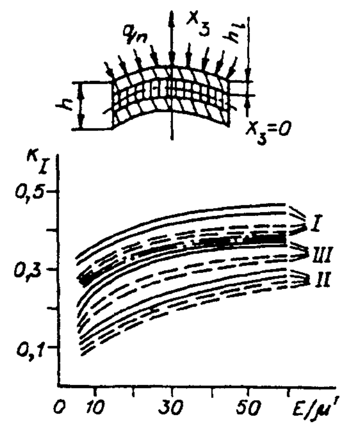

Fig. 2

Two methods are used for the numerical determination of the intensity G_I or the factor K_I: the differential-stiffness method (the compliance method), and the method of virtual growth of the crack, described in [1, 10]. The main difficulty in both these methods is calculating the components of the displacement vector. The finite-element method in the displacement version is used to determine them. The main problem in the finite-element method is deriving the stiffness matrix [11]. Using the method described in [12, 13] we obtained the stiffness matrix of a triangular finite element of nonzero Gaussian curvature of stratified orthotropic material for the Timoshenko-type geometrically and physically nonlinear theory. The stiffness matrix obtained is also used to calculate stratified transversely isotropic and isotropic arbitrary shells with slit-cracks [1].

3. Numerical Examples and Their Analysis. We will consider a three-layer isotropic square freely supported cylindrical panel with three cases of the arrangement of the slit-crack, namely, I) at the center, II) from the side, and III) collinear with both sides. The geometrical and physical characteristics of the panel, shown in Fig. 1, are as follows: h = 0.01 m, R = 0.20 m, a = 0.30 m, f = 0.0677 m, and $0 \leq \nu \leq 0.5$. Figure 1 shows graphs of K_I as a function of L/a. For all situations of the position of the slit-crack the semiaxis of symmetry of the panel was divided into 13 and 25 nodes. When the semiaxis of the panel was divided into 13 nodes the factor K_I was calculated using the compliance method, and when it was divided into 25 nodes it was calculated using the virtual growth of the crack method (this procedure confirms the convergence and accuracy of the numerical results). The relative error between the graphs obtained when the semiaxis of symmetry was divided into 13 and 25 nodes, for all cases of the arrangement of the slick-crack, does not exceed 6%. The continuous curves are graphs obtained using the geometrically and physically nonlinear theory of shells (for noncontacting sides of the slit, i.e. with $c = 8 \times 10^{-5}$ m, which was found by the method described in [14]), and dashed curves were calculated using the physically nonlinear theory. We can conclude from an analysis of the corresponding graphs that contact between the sides of the slit has only a slight effect on K_I (the error is less than 5%).

For the above cylindrical panel, but from the transversely isotropic layers, Fig. 2 shows graphs of K_I as a function of E/μ' (where μ' represents the transversality of the panel over the thickness) for all situations of the position of the slit-crack for h/L = 0.5. The method of dividing the semiaxis of symmetry of the panel and the methods of determining K_I are similar to those in the previous case, and the notation in the graphs corresponds to that shown in Fig. 1. The graphs in Fig. 2 illustrate the effect of transversality of the shell over the thickness on K_I for all types of theory. The relative error between the corresponding graphs for all situations of the arrangement of the slit-crack is not greater than 6%.

In exactly the same way as in the previous case, we can consider a panel whose layers possess the properties of an orthotropic material with a slit-crack at the center, and determine the effect of the degree of orthotropy $E_{x_1}{}^i/E_{x_2}{}^i$ on K_I. The results for the geometrically and physically nonlinear theory are shown in the table ($h/L = 0.5$, the upper numbers were obtained for $n = 13$ and the lower numbers for $n = 25$), from an analysis of which we can conclude that the degree of orthotropy has no effect on K_I. The relative error between the calculations is due to computational errors and the effect of the density of the grid.

We can conclude from the above results that the very nonlinear variational problem of the Timoshenko-type theory of anisotropic stratified shells containing shift-cracks formulated above can be solved fairly simply using the energy approach.

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